Derivation of a macroscopic model for two-phase non-Darcy flow in homogeneous porous media using volume averaging

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Abstract:

The purpose of this work is to propose a derivation of a macroscopic model for a certain class of inertial two-phase, incompressible, Newtonian fluid flow through homogenous porous media. The starting point of the procedure is the pore-scale boundary value problem given by the continuity and Navier–Stokes equations in each phase β and γ along with boundary conditions at interfaces. The method of volume averaging is employed subjected to a series of constraints for the development to hold. These constraints are on the length- and time-scales, as well as, on some quantities involving capillary, Weber and Reynolds numbers that define the class of two-phase flow under consideration. The development also assumes that fluctuations of the curvature of the fluid–fluid interfaces are unimportant over the unit cell representing the porous medium. Under these circumstances, the resulting macroscopic momentum equation, for the α-phase (α=β, γ) relates the gradient of the phase-averaged pressure $\nabla\langle p_\alpha\rangle^\alpha$ to the filtration or Darcy velocity $\langle \mathbf{v}_\alpha\rangle$ in a coupled nonlinear form explicitly given by

$$\langle \mathbf{v}_\alpha\rangle = -\frac{\mathbf{K}^*_{\alpha\alpha}}{\mu_\alpha}\cdot\left(\nabla\langle P_\alpha\rangle^\alpha - \rho_\alpha \mathbf{g}\right) - \mathbf{F}_{\alpha\alpha}\cdot\langle \mathbf{v}_\alpha\rangle$$

$$-\frac{\mathbf{K}^*_{\alpha\chi}}{\mu_\chi}\cdot\left(\nabla\langle P_\chi\rangle^\chi - \rho_\chi \mathbf{g}\right) - \mathbf{F}_{\alpha\chi}\cdot\langle \mathbf{v}_\chi\rangle \qquad \alpha,\chi = \beta,\gamma \;\; \alpha \neq \chi$$

In these equations, $\mathbf{F}_{\alpha\alpha}$ and $\mathbf{F}_{\alpha\kappa}$ are the *inertial* and *coupling inertial correction tensors* that are functions of flow-rates. The dominant and coupling permeability tensors $\mathbf{K}^*_{\alpha\alpha}$ and $\mathbf{K}^*_{\alpha\kappa}$ are intrinsic and are those defined in the conventional manner. All these tensors can be determined from closure problems that are to be solved using a spatially periodic model of a porous medium. Some indications to compute these tensors are provided.